Jan Evangelista Purkyně University in Ústí nad Labem
Faculty of Science
Department of Mathematics

Abstract - Doctoral Thesis

Development of Logical Thinking Using Mathematical Games

by
Mgr. Vlastimil Chytrý

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Rozvoj logického myšlení pomocí matematických her

Mgr. Vlastimil Chytrý

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Doktorand: Mgr. Vlastimil Chytrý

Školitel: Prof. RNDr. Jiří Cihlář, CSc
Katedra matematiky, Přírodovědecká fakulta, UJEP, Ústí nad Labem
Klíšská 30, 400 01 Ústí nad Labem

Oponenti: Prof. RNDr. Josef Molnár, CSc.
Katedra algebry a geometrie, Přírodovědecká fakulta, UP, Olomouc
17. listopadu 12, 771 46 Olomouc

Doc. RNDr. Naďa Vondrová (Stehlíková), Ph.D.
Katedra matematiky a didaktiky matematiky, Pedagogická fakulta, UK, Praha
M.D. Rettigové 4, 116 39 Praha 1

Autoreferát byl odeslán dne_____

Obhajoba se koná dne _____ v _____ hodin před komisí pro obhajoby disertační práce v oboru P1101 Obecné otázky matematiky na Přírodovědecké fakultě Univerzity J. E. Purkyně v Ústí nad Labem, místnost _____ + adresa_____.

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Předseda OR: Doc. RNDr. Miroslav Lávička, Ph.D.
Katedra matematiky, Fakulta aplikovaných věd, ZČU, Plzeň
Univerzitní 22, 306 14 Plzeň

Místopředseda OR: Doc. PaedDr. Petr Eisenmann, CSc.
Katedra matematiky, Přírodovědecká fakulta, UJEP, Ústí nad Labem
Klíšská 30, 400 01 Ústí nad Labem

Introduction

Teachers at both primary and secondary schools often claim, that they are trying to develop logical thinking with their pupils which Hartl and Hartlova (2010) describe as ‘a higher form of thinking than the activity-dependent one, i.e. the correct reasoning following the laws of formal logic, where the two basic approaches are distinguished: the derivation from the general to the speific, i.e. the deduction, and the derivation from the specific towards the general, i.e. the induction.‘ Having got acquainted as a teacher with the instruction at primary and secondary level over the period of several years, I have found out that some teachers develop rather mathematical thinking, defined by Dunlap (2010) as ‘a cognitive approach to a problem, which is of both logical and mathematical character.‘

By analysing the instruction of mathematics in the classes observed I have come to the conlusion that the instruction is often focused on mastering a particular algorithm, only suitable for solving the model tasks. This way of tuition led to a situation where the pupils were able to solve a task, but could not justify the procedure. When working with altered assignment, the pupils often did not know how to solve the task, unable to react to the change. Such instruction surely does not develop logical thinking. Having interviewed both primary and secondary teachers I assume, that they do not distinguish between the two types of thinking, and usually do not consistently develop logical thinking with their pupils.

The logic can be approached in two ways. Within the first (psychological) approach, the logic deals with thinking processes leading an individual to particular conclusions. It admits that the person makes use of common language, while the concequences are also inferred on the basis of one´s own experience. Thus the disjunction can be understood in the sense of elimination, the implication is based on factual meaning of statements etc. Commonly the method of blind attempts is employed, which should not have place in the world of logic (Peregrin, 2004).

The second concept considers the logic as a formal science, studying the ways of drawing conclusions from pre-defined premises. Frege and other thinkers, the founders of formal logic, arrived at the conclusion that even the structure of language and our reasoning should be isolated and ‘mathematicised‘, so that they could be analysed through already verified means of mathematics. This brings partial reglementation to the matter (Peters, 1998).

With regard to the fact that authors do not agree on the definition of logical thinking, I have suggested my own concept, which considers as essential the ability of abstraction and reasoning.

Amongst the authors who deal with the issues of abstraction and reasoning are also Graham, Cuoco and Zimmerman (2010). They describe in their work the individual´s considerations employed in mastering algebra, as well as the best way to proceed towards algebraic notation. One chapter of their book is focused on using symbols, i.e. the abstraction lift in the transition from a constant to a variable.

In this work I have focused not only on the relationship between ontogenesis and phylogenesis in the field of logic, but mainly on the quantitative research combined with the qualitative one.

One of the targets of the work is to carry out, in the form of pedagogical research, the analysis of the present level of logical thinking with pupils of primary and secondary schools. We can presuppose that similarly to the intelligence of an individual, the logical thinking will show relative stability, though some researches mention the possibility of positive influence on the ability of abstraction and logical reasoning (Lee, 1990). Another aim is then the answer to the question whether a positive shift is possible in the fields focused on the ability of abstraction and the ability of reasoning through the use of mathematical and logical games.

The Characteristics of Logical Thinking

Logical thinking

When exploring the term of logical thinking, we do not find an unequivocal definition. Dr. Karl Albrecht (1984) claims that the base of logical thinking is sequential thinking. This process involves the acceptance of important ideas, facts and conclusions which are part of problems, and ranges them into a progressive chain that carries the importance of its own. Thus, it is only possible to think logically within the steps that do not contradict one another. Contrary to this, Labouvie (1992) maintains that with logical thinking the important fact is the complex solution to a problem, requiring not just the survey of systematic logical thinking through the characteristics of formal operations, but also the choice and the interpretation of the particular area from which the logical thinking arises. There exist more such definitions described in the work, and I find it difficult to simply adopt one of them. I have thus decided to specify my own definition of the notion of logical thinking.

The Author´s Concept of Logical Thinking

On the basis of the individual approaches published I conclude, that it is not possible to exactly define and consequently ´measure´ logical thinking as a whole, but it is necessary to narrow the issue and establish the most significant attributes of logical thinking.

I suggest that **logical thinking is a proces in which the individual looks back from the content of particular statements, and consistently employs particular inferences, so that he arrives at a correct conclusion. The indisputable partial steps of the process create a link between the assumptions and the conclusion through a chain of these inferences.**

I also suppose that logical thinking is part of the mathematical one, while the mathematical thinking presupposes extra knowledge of particular mathematical factors, formulas, algorithms etc.

With the construction of the test for measuring the level of logical thinking, I focus on the ability of abstraction and correct inference. Given that the pupils face abstraction not only within mathematics, but also geometry, I have incorporated in the test the items that aim at seeking numerical or geometrical regularities. My test (supplement 1), though called simply ´the test of logical thinking´, focuses namely on the ability of abstraction and the ability of correct reasoning.

The Method of Research

The research issues, aims and hypotheses

The research described in this doctoral thesis focuses namely on two research issues, the first being the description of the present level of logical thinking with pupils of different ages, and the factors that influence this level. The second issue of the research is the survey in the possibility to influence positively the level of logical thinking through medium-term application of suitably chosen mathematical or logical games.

For solving the research issues it is first necessary to find the method of the logical thinking level´s detection, then, to define the possible external factors which influence it, and last but not least, to select suitable mathematical or logical games and describe the way to measure an individual´s capacity to play them. That is why I have established for the purposes of this work the following targets:

1. Selecting on the basis of suitable criteria the games that develop logical thinking with pupils, and specifying the benefits of their implementation to the instruction of mathematics.
2. Finding and describing the factors influencing the effectiveness of logical games´

incorporation in teaching mathematics (namely the individual IQ, the school´s evaluation and the type of school).

3. Drawing up a test assessing the level of logical thinking of pupils.
4. Finding out what factors have impact on pupils´ logical thinking.
5. Finding out whether positive shifts in logical thinking of pupils are monitored after medium-term application of logical games in teaching mathematics.

One of the factors that may affect the level of logical thinking is the age of an idividual. Had I chosen more values for the variable of age (e.g. 10 to 18), the extent of the selective set would not have been proportional with regard to the interaction with the other factors. For that reason I have limited the choice into two categories, i.e. the younger pupils (primary school pupils) and the older pupils (secondary school pupils), and have introduced the variable of school type.

For the complexity of the structure of the whole research I present the following chart, with the help of which the basic hypotheses have been established.

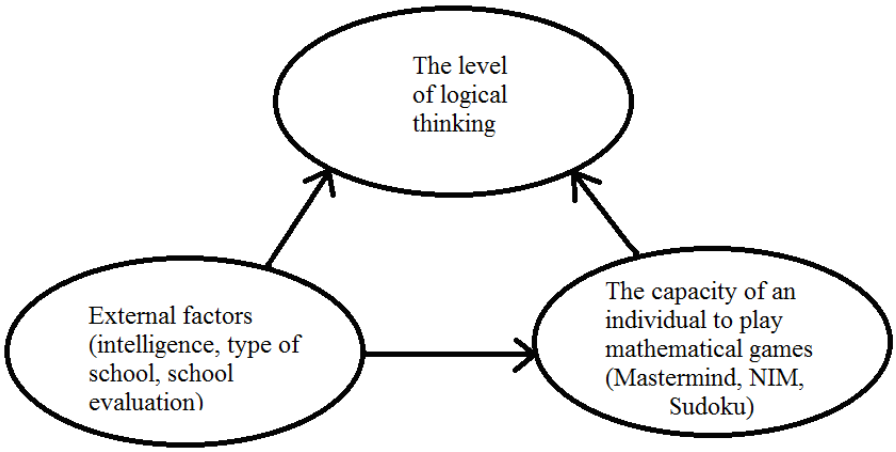


Chart characterising the dependences observed

Apart from the dependeces between the particular domains of the chart, possible dependences have also been searched within the particular domains themselves, as suggested in the following hypotheses:

H_1: Amongst the external factors (intelligence, type of school, school evaluation), the predominant part is taken by the intelligence of an individual.

H_2: The level of logical thinking of an individual depends on external factors (intelligence, type of school, school evaluation).

H_3: The capacity of an individual to play mathematical games (Mastermind, NIM, Sudoku) depends on external factors (intelligence, type of school, school evaluation).

These basic hypotheses are further divided into subsequent hypotheses, e.g. the hypothesis H_3 falls into nine partial hypotheses etc. With regard to the last research target, I shall define yet another basic hypothesis:

H_4: Through medium-term influence on pupils by means of the application of mathematical and logical games, it is possible to reach positive changes in their logical thinking.

For the verification of this last hypothesis, the level of logical thinking must be measured in two time periods (pre-test and post-test).

The Research Project Description

A range of research methods and techniques have been employed in the work. Apart from the content analysis of basic pedagogical documents, teaching experiments have been used focusing on logical games, the method of questionaire for collecting mass data, and the technique of directed individual interviews. The interviews were carried out within each part of the research, where it was not evident from the questionaire which considerations the pupil engaged, or when it was necessray to answer questions that could not be incorporated in the questionaire. Case studies were carried out so as to assess the shift in the level of pupils´ logical thinking and logical expression.

The selection of respondents

The respondents were chosen by multilevel random selection. From the basic category (all primary or secondary pupils over the age of eleven), twenty schools were drawn in the region of Ústí nad Labem, which were then addressed and asked for cooperation. That differed in compliance with the character of research. When it was not necessary to stay in the school longer than a few hours (assigning the test of intelligence and the test of logical thinking), there were more pupils assembled than in the case where 10-12 hours of presence at school were required (selection of the respondents for the experiment assessing the effectiveness of the application of games in teaching). With the latter it was not only necessary to address the pupils in the experiment, but also those in the the control groups. The following table shows the numbers of cooperating respondents. It presents the pupils who did not play games in the experiment as ’the others‘. Those only passed the tests of intelligence and logical thinking.

	Primary school	Secondary school	Total
Experimental group	60	48	108
Control group	59	47	106
The other pupils	88	127	215
All respondents	207	222	**429**

Numbers of respondents

Tests of intelligence

Testing intelligence is basically possible in two ways.

The first way makes use of complex tests of intelligence, which test different abilities (the test Army alfa and Army beta, Wechsler´s tests, the test of the intelligence structure, the analytical test, Snijders-Oomen´s test, tests MIT, TURS, KAI etc.). These tests are dealt with by specialised advisory centers and clinics. An individual examination takes approximately 60-90 minutes, and it is necessary for it to be carried out by a professional psychologist. Such tests are realised with regard to sorting particular problems of a client, e.g. the selection of school, job etc. Similar results would undoubtedly be a contribution to my work, though, it is not possible to carry out this testing with over 400 respondents.

The second way employs partial tests of intelligence focused on general intelligence, namely the Spearman´s G-factor. Amongst the most frequently used tests belong primarily the test of progressive matrices (J.C.Rawen), the CF 1, CF 2, CF 3 tests (Cattell), the Domino test, Kohs´ cubes etc. Considering the character of the work and time possibilities, the Rawen´s test of progressive matrix has been employed, which allows testing within a teaching unit.

The questionaire research and its statistic processing

The footing for evaluating the level of logical thinking was obtained through printed quetionaires. It was not possible to make use of the existing questionaires or tests of logical thinking, as those do not reflect the level of logical thinking in the way I had adopted for the purposes of this work. A new original test of logical thinking was thus drawn up (see the supplement) in two versions; once as a pre-test, then as a post-test.

Each logical test included 12 main items, some of which were further segmented. On the whole, each respondent was solving 22 items, which could be divided into three domains:

- Search for Numeral Regularities (ability of abstraction) – NR
- Search for Geometrical Regularities (ability of abstraction) – GR

- Ability to draw Correct Conclusions – ACC

Each of these domains contains the same number of items. The answers to the questions with the individual items have been evaluated alternatively:

- 0 – the pupil answered incorrectly,
- 1 – the pupil answered correctly.

If the pupil did not respond to a question, an empty sign was used for the coding. Such way of coding allows this interpretation of results: the arithmetic mean of the values measured is the suitable point estimation of the *p* parameter of the alternative distribution, which is the probability that a randomly chosen pupil will answer the related question correctly.

This way allows to evaluate the level of logical thinking of an individual by means of the evaluating vector in the form (NR, GR, ACC). If a pupil has the evaluating vector of (0.73; 0.20; 0.50), then the first component of the vector reports on the fact that the pupil can for example respond correctly to a question from the field of searching for numeral regularities with the probability of 73%. In my research I have made use of this particular type of logical thinking evaluation, as it allows not just the assessment of the progress or decline as such, but also the evaluation of the individual parts in which the change has occured.

For the examination of the dependences of the answers with the individual items in the questionaire the Spearman´s correlation coefficient of ordinal correlation has been employed.

Reliability of testing

The evaluation of the reliability of the test of logical thinking has been carried out on the basis of the Split–half reliability test, where the test items are divided into two approximately equal parts. In this case I have divided each of the researched domains into two parts, as illustrated in the following table:

Split-half reliability	First Part			Second Part		
Examined domains	NR	GR	ACC	NR´	GR´	ACC´
Items of the given domain	1A, 3, 5B,	6A, 6C,2A	7C, 7B, 11, 12	1B, 1C, 5A	2B, 4, 6B, 9	7D, 7A, 8, 10B, 10A

Division into items of the logical thinking test with the Split – half reliability

For this division I have counted out the evaluating vector, separately for the first and the second part with each respondent. The accordance of these two partial evaluating vectors has

been considered according to the mutual correlation of the individual items which have reached the value of 0.59 for NR, 0.54 for GR and 0.58 for ACC. The disadvantage of the subsantial lowereing the correlation coefficient with the Split–half reliability test is corrected by using the Spearman-Brown formula, which sets the reliability for the whole of the test unshortened. The formula has generally the following form:

$$r_{x,x'} = \frac{m \cdot r'_{x,x'}}{1+(m-1)\cdot r'_{x,x'}},$$

where *m* is the ratio of the number of the original test´s questions to the number of questions in the shortened test; here *m* = 2. For example, with the domain of seeking numeral regularities we can thus count out:

$$r_{NR,NR'} = \frac{m \cdot r'_{NR,NR'}}{1+(m-1)\cdot r'_{NR,NR'}} = \frac{2 \cdot 0.59}{1+1 \cdot 0.59} = 0.74.$$

The values of the other parts are 0.70 for GR and 0.73 for ACC. On the basis of Klin´s principle, which considers a test reliable on condition that the levels are at least 0.70, the testing method can be considered reliable enough (accessible at: http://cs.wikipedia.org/wiki /Reliabilita). With regard to the fact that our division of the responses into the items of the test of logical thinking is of dichotomic character, it is possible to make use of yet another method of reliability assessment. This procedure employs the Kuder – Richardson formula. The authors introduced more formulas, while I have only used the formula no. 21 (accessible at:http://www.e-metodologia.fedu.uniba.sk/index.php/kapitoly/ziskanie-hodnotnych-dat/zistovanie-reliability.php?id=i9p3), whose general form is as follows:

$$KR_{21} = \frac{K}{K-1} \cdot \left[1 - \frac{AP \cdot (K - AP)}{K \cdot S^2}\right],$$

where *K* represents the number of items within the examined domain (e.g. *K*=6 with searching the numeral regularities), AP is the arithmetic mean of the individual values obtained in the particular domain, and S^2 is the variance of the given sample. When, for example, incorporated into the formula for searching geometrical regularities, we gain the following:

$$KR_{21} = \frac{7}{7-1} \cdot \left[1 - \frac{4.79 \cdot (7 - 4.79)}{7 \cdot S^2}\right] = 0.51,$$

The values of the other parts are 0.53 for NR and 0.57 for ACC. Though these values are lower than those obtained with the first method, we can consider the results between 0.5 and 0.6 as satisfactory.

To define the relationships between the individual items of the evaluating vector, I have made use of Spearman´s Correlation Coefficient of Serial Correlation, as illustrated in the following table.

Examined Domains	NR	GR	ACC
NR	1.00000	0.58436	0.540258
GR	0.584362	1.00000	0.529890
ACC	0.540258	0.529890	1.00000

Values of correlation coefficients with the examined dependences

It is highly positive and important for this work, that all the constituents of the evaluating vector show substantial mutual correlation. The strong correlation between the components evaluating the search for dependences is not surprising, but the confirmed link between the domain focused on the ability to draw correct conclusion, and the other two domains.

The Statistic Methods Employed

In the research, the following methods have been used for obtaining data and their further processing. During the testing I have gained data of different character. It was necessary to differntiate between dependent and independent selections, nominal, ordinal and metric random quantities, with the metric quantities to evaluate their normality and according to that, to opt for parametric or non-parametric statistic methods. The following statistic methods and techniques have been used in course of the research:

- testing normality (Shapiro Wilcoxon normality test, K-S Lilliefors test for normality),
- non-parametric testing of hypotheses (Mann-Whitney test, Wilcoxon paired test),
- non-parametric analysis of variance (Kruskall-Wallis test),
- correlation analysis (Spearman´s rank correlation coefficient),
- methods of measuring dependences between quantitiative and qualitative variables,
- independence tests (contingence tables),
- cluster analysis,
- methods of reliability assessment (split-half reliability, Kuder-Richardson formula no. 21).

Teaching experiments with logical games

When searching for suitable games I considered several basic features for such games to carry:

- the use of abstraction and correct conclusion drawing is essential when playing the game,
- the game is not demanding material-wise,
- the game in its form is applicable in teaching,
- the game does not involve complicated rules.

The individual criteria and the choice of the games are futher described in this work; for the experimnent they were the games of Sudoku, Mastermind and NIM.

It was also important for the selection of the games how exacting conclusions were employed in their course. With the game of Sudoku only simple conclusions are required, based on the characteristics of conjunction, disjunction and implication. With the game Mastermind more complex inferences are involved, and namely their sequencing for preserving the consistence of the individual moves. While with the games of Sudoku and Mastermind the knowledge of the prepositional calculus is required, the first-order logic (work with quantifiers) is only employed with the game of NIM, namely when defining the win and lose positions.

The incorporation of the games in the school classes was carried out in the following combinations (this approach was chosen for time reasons, as it was not possible to introduce all the games to the entire number of classes):

- NIM and Mastermind,
- Sudoku and Mastermind,
- Sudoku and NIM.

After the presentation of rules, the frontal demontration of the game followed, and questions upon the rules were answered. Consequently, the answer sheets were distributed among the pupils, including the assignment and the spaces for inserting the responses reflecting the progress of the game. When time measuring was necessary, a stopwatch was displayed on the interactive board. The individual abilities to play the chosen games have been assessed differently. The particular answers were encoded by nominal or ordinal quantities, while the individual´s ability to play the games was evaluated alternatively by the codes of 0 or 1. The actual coding is as folows:

- with the game of Sudoku it was observed whether the whole table was filled in correctly,
- with the game Mastermind it was evaluated whether the pupil had made use of all the information from the previous moves,
- with the game NIM it was assessed whether the pupil had discovered the winning strategy or not.

So as to make it possible to keep record of the individual´s abilities to play the games, it was necessary to create a record sheet for each game. A sample of the sheet is included in supplement 2. The whole experiment was accompanied by interviews with pupils, which clarified their steps and ways of searching strategies. On the basis of these interviews I observed the relationship between the language of an individual and his abilities of reasoning (namely when working with quantifiers).

The Obtained Results

In this part we shall focus namely on the following:

- the description of the external factors (IQ, type of school, school evaluation in mathematics) and their mutual relationships,
- the influence of the external factors on the level of logical thinking,
- the description of the experiment group and the control group and their comparability,
- the assessment of the level of game playing,
- the influence of the external factors on the ability to play the selected games,
- progress in the level of logical thinking.

The External Factors (IQ, type of scool, the school evaluation in mathematics)

I describe in my thesis the research in the dependences between the external factors which I consider significant as for the influence on the level of logical thinking. It is not surprising that the dependence between the school evaluation and the type of school has not been proved, similarly to the expected fact that the IQ of secondary school students is higher than that of the primary school pupils. With both the primary and secondary schools, the dependence has been proved between the school evaluation and the intelligence of the pupils.

The impact of the external factors on the level of logical thinking

The matter of influence of the external factors on the level of logical thinking is discussed with regard to the levels obtained in the introductory test of logical thinking. We can say that the level of influence on the level of a pupil´s logical thinking grows in accordance with the level of his IQ itself. The influence of the school evaluation on the level of logical thinking has been confirmed with both primary and secondary grammar schools. It has been proved that the levels of logical thinking with the pupils of primary schools and those of ‘the other secondary schools‘ do not differ extensively from one another, whereas the higher IQ level of the secondary grammar school pupils´ shows a statistically significant shift in the level of logical thinking, when compared with the pupils of the other two types of schools.

The Experiment Group and the Control Group and their Comparability

Considering the orientation of the experimental part of this work, both the control and experiment group at the two types of school have been compared namely in the domain of the level of logical thinking. The following table shows the numbers of pupils whose level of logical thinking has been examined.

	Primary School	Secondary School
Experiment group	60	48
Control group	59	47

Numbers of respondents in course of the experiment

The numbers of the pupils in the experiment and the control group are roughly comparable.

The control group with the primary school pupils can be considered as suitable, as its pupils reach approximately the same values in the pre-test of logical thinking as with the pupils of the experiment group.

We did not manage, though, to ensure for the experiment and the control group within secondary schools to be comparable in that sense. The group where the experiment with logical games was carried out belonged to the category of ‘other secondary schools‘, while the intended control group recruted from the grammar schools. The control group is thus unsuitable for comparison. As a consequence, it is not possible to compare the results of both the two groups using the statistical methods for secondary schools.

The assessment of the game playing level

During the research focused on the ability of primary and secondary school pupils to play the chosen games, I have arrived at several partial conclusions related to the interest of pupils in these games. With the game of Sudoku, whose rules are commonly known by general public, no substatial difference has been proved in the interest of primary and secondary pupils, nor in the ability to play the game. With the game NIM, the interest varied from one phase of the exeriment to another. The primary school pupils showed more interest in the game where that involved manipulation with objects, while the secondary students seemed more interested when the game, aimed at discovering the winning strategies, made use of abstraction and graphs. The game Mastermind proved more attractive with secondary school pupils.

The impact of external factors on individual´s ability to play the selected games

In this chapter I focus on the ways the observed factors influence the ability of an individual to play the games, when these are incorporated in lessons. Based on the data obtained in the experiment group the following table has been assembled, describing what dependences have been proved between the particular factors and abilities at the significance level of 5 %.

Types of games/individual factors	Type of school	Individual´s intelligence	School evaluation
Individual´s ability to play Sudoku	No	Yes	No
Individual´s ability to play Mastermind	Yes	Yes	No
Individual´s ability to play NIM	Yes	No	No

Numbers of respondents in course of the experiment

Only with the game of Sudoku the dependency on the type of school has not become evident. This is not surprising, given that the ability to play this game is namely defined by the frequency of playing, as it has been mentioned above. What is surprising is the fact that the dependency which has been proved between the individual´s intelligence and the ability to play the games of Sudoku and Mastermind has not become evident with the game of NIM. Nor is it particularly unexpected that the dependency has not been proved between the ability to play the games and the school evaluation.

Progress in the level of pupils´ logical thinking

As it has been stated, I have focused in my work namely on the development of an individual´s logical thinking with the help of mathematical or logical games. For the purposes of the final testing it was necessary to create an end test of logical thinking, that would be analogous to the entry test of logical thinking. The following table provides the numbers of respondents in both the experiment and the control group for primary and secondary level. Only those respondents are included, who have also sat the pre-test and the post-test of logical thinking.

	Primary school	Secondary school
Experiment group	41	31
Control group	65	38

Numbers of respondents in the course of the experiment

This part of research is related to the hypothesis claiming that through the influence on pupils by means of medium-term application of mathematical and logical games, it is possible to reach positive changes in their logical thinking. The verification of this hypothesis has been carried out sepatately with primary and secondary school pupils.

Primary school

The hypothesis has been proved with the primary school experiment group in all the examined domains. To confirm the fact that the positive changes in the level of logical thinking occured right on the basis of the incorpotration of the selected games in lessons, I carried out analogous testing with a primary school control group. Here, the hypothesis has not been proved in any of the examined domains.

Secondary school

The hypothesis has been proved with the secondary school experiment group in all the examined domains. As it is problematic to make use of a control group (see above), these results can only be considered as approximate. Nevertheless, I state that with the control group the hypothesis has not been proved in any of the examined domains.

It is remarkable that though the games were played with the pupils in only ten lessons, there was a statistically significant improvement observable in all the examined domains (with the exception of searching for numerical regularities with primary school pupils). It is thus possible to confirm the hypothesis H_4, that through medium-term influence by means of

the application of mathematical and logical games with pupils, it is possible to reach positive changes in their logical thinking.

Conclusions

My pedagogical research, including all the preliminary phases dealing with mapping the environment and the popularity of the games, was carried out at nine schools, where the total number of 678 respondents were addressed. Substantial part of the research was a teaching experiment aimed at the incorporation of games in lessons; there I met 108 pupils in thirteen sessions. Collecting the experiment data was realised through the method of questionaire, with the help of entry and end tests of logical thinking, the intelligence test and several answer sheets. Specific areas of the research were accompanied by individual directed interviews with chosen respondents. On the basis of these interviews I came to the conclusion that an important condition for drawing correct logical conclusions is using suitable language.

Through the observations in class and the interviews with teachers and pupils, I have found out that there is a generally accepted belief, that logical thinking of pupils is developed rather automatically, i.e. by learning Mathematics itself. My own research has led me to the conclusion that this is only partially so, and that more attention to the development of logical thinking should be paid. While the Framework Education Programme (henceforth the FEP) for secondary schools mentions the basic topics of logics explicitely as part of the chapter Argumentation and verification, there is hardly any space given to the topics of logics in the FEP for primary education (accessible at:
http://www.msmt.cz/vzdelavani/skolskareforma/ramcove-vzdelavaci-programy).

Another interesting finding obtained from the teaching experiment is the fact that while with the primary school pupils the ranking of games from the least to the most popular one is the Mastermind, the Sudoku, the NIM, with the secondary school students the order is reversed. As a possible explanation I suggest that with primary pupils the possibility of manipulation with objects involved played a positive role, whereas with secondary pupils this aspect was perceived rather as an obstacle. Some of the pupils also played the selected games as their leisure time activity.

Throughout the experiment it was constantly heeded to create positive educational environment. Helmke (2007) presents ten basis features essential for running the educational process, including namely the pupil´s motivation and activation, the structure and clarity of the instruction, individualization, cohesiveness, focus on the pupil, both teacher´s and pupil´s feedback, utilization of mistakes in the pupil´s cognitive processes etc. When playing games,

the pupil is motivated and activated in a natural way, ech playing one´s own part while having one´s competitor (except for Sudoku), while getting the immediate feedback on one´s performance, naturally making mistakes and eliminating them on one´s own immediately etc. The teaching environmnent created in the experiment thus supported the self-fulfillment of the pupils, their individual work, and it respected their individual pace. It can also be stated that when playing these games, the pupil goes through all the stages of the cognitive process.

With the pupils of the experiment group at primary school level I have found, when comparing the pre-test and the post-test, the statistically significant difference in the level of their logical thinking in the domains of searching for numerical regularities and the ability of drawing right conclusions at the level of significance of 5 %, and with the geometrical regularity at the significance level of 1 %. With the pupils of the control group for primary schools, no statistically significant difference has been found in any of the examined domains.

With the pupils of the experiment group at secondary school level, a statistically significant difference has been found in the level of their logical thinking within all the examined domains at the significance level of 1 %. In the group of secondary school pupils where the experiment was not carried out (it is not possible to take it as a standard ‘control group‘ for the reasons mentioned above), I have not found a statistically significant difference in the level of logical thinking between the pre-test and the post-test in any of the examined domains. It can thus be stated that the hypothesis H_5 has been proved. I thus infer that it is possible to positively influence the level of logical thinking (abstraction and reasoning) of an individual.

These conclusions of mine correspond with the research carried out by Lee (1990). He describes abstraction as integral part of individual´s intelligence, and claims that it is possible to influence it positively. Lee further deals with the researh in the ability of human abstraction with regard to one´s age, and he arrives at the conclusion that along with the age it slightly decreases. He also mentions that the relationship between an individual´s intelligence and the ability of one´s abstraction is invariable throughout one´s life. The strong dependence of an individual´s ability of abstraction on one´s intelligence has also been confirmed in this doctoral thesis.

On the basis of my research, I have become convinced that the use of similar methods would also bring its benefit at universities, namely for the students who prepare for the career of a teacher.

Závěry

Můj pedagogický výzkum, včetně všech předvýzkumů se zaměřením na zmapování prostředí a oblibu her, proběhl na devíti školách, kde bylo osloveno celkem 678 respondentů. Podstatnou součástí výzkumu byl didaktický experiment zaměřený na zapojení her do vyučování, kde jsem se setkal se 108 žáky celkem třináctkrát. Sběr experimentálních dat byl proveden dotazníkovou metodou pomocí vstupního a výstupního testu logického myšlení, testu inteligence a několika záznamových archů. Specifické oblasti výzkumu byly doplněny řízenými individuálními rozhovory s vybranými respondenty. Na základě těchto rozhovorů jsem také došel k závěru, že používání vhodného jazyka je důležitou podmínkou pro správné logické usuzování.

Na základě pozorování v hodinách a rozhovorů s učiteli a žáky jsem zjistil, že je široce rozšířen názor, že k rozvoji logického myšlení žáků dochází víceméně automaticky tím, že se učí matematice. Z vlastních šetření jsem došel k závěru, že je tomu tak jen částečně, a že ve výuce matematiky je třeba rozvoji logického myšlení věnovat větší prostor. Zatímco Rámcový vzdělávací program (dále jen RVP) pro střední školy explicitně zmiňuje základní logické učivo, v části Argumentace a ověřování, RVP pro základní vzdělávání nevěnuje logickému učivu téměř žádný prostor (dostupné z http://www.msmt.cz/vzdelavani/skolska reforma/ramcove-vzdelavaci-programy).

Zajímavým poznatkem z didaktického experimentu bylo zjištění, že zatímco u žáků základní školy je pořadí od nejméně po nejvíce oblíbenou hru Mastermind, Sudoku, NIM, u žáků ze střední školy je toto pořadí obrácené. Toto vysvětluji tím, že u žáků základní školy hrála pozitivní roli možnost doprovázet hraní hry konkrétní manipulací s předměty, kdežto u žáků středních škol byl tento fakt spíše překážkou. Někteří z žáků se vybraným hrám věnovali i ve svém volném čase.

V experimentu bylo důsledně dbáno na vytvoření příznivého výukového prostředí. Helmke (2007) uvádí deset základních charakteristik, které jsou podstatné pro vedení výuky. Patří mezi ně zejména motivace žáka a jeho aktivizace, strukturovanost a jasnost výuky, individualizace, soudržnost, orientace na žáka, zpětná vazba učitele i žáka, využití chyby v žákově poznávacím procesu apod. Při hraní her je žák motivován a aktivizován přirozenou cestou, každý hraje sám za sebe a zároveň má svého soupeře (vyjma hry Sudoku), získává ihned zpětnou vazbu o svém výkonu, přirozeně se dopouští chyb, které vzápětí sám odstraňuje atd. V experimentu vytvořené výukové prostředí tak podporovalo seberealizaci žáků, jejich

samostatnou práci a respektovalo jejich vlastní tempo. Je také možné konstatovat, že při hraní těchto her žák prochází všemi stupni poznávacího procesu.

U žáků experimentální skupiny základní školy jsem při srovnání pretestu a posttestu shledal statisticky významný rozdíl v úrovni jejich logického myšlení v oblastech hledání číselných zákonitostí a schopnosti správného úsudku na pětiprocentní hladině významnosti a v případě geometrické zákonitosti na jednoprocentní hladině významnosti. U žáků kontrolní skupiny pro základní školy nebyl shledán statisticky významný rozdíl ani v jedné ze zkoumaných oblastí.

U žáků experimentální skupiny střední školy byl shledán statisticky významný rozdíl v úrovni jejich logického myšlení ve všech zkoumaných oblastech na hladině významnosti jednoho procenta. Ve skupině žáků střední školy, kde experiment neprobíhal (nelze ji korektně považovat za kontrolní z důvodů výše uvedených) jsem neshledal statisticky významný rozdíl v úrovni jejich logického myšlení v pretestu a posttestu v žádné ze zkoumaných oblastí. Lze tedy konstatovat, že hypotéza H_5 se potvrdila. Domnívám se tedy, že je možné pozitivně ovlivnit úroveň logického myšlení (abstrakce a usuzování) jedince.

Tyto mé závěry se shodují s výzkumem, který provedl Lee (1990). Ten popisuje abstrakci jako nedílnou součást inteligence jedince a konstatuje, že je možné ji pozitivně ovlivnit. Lee se nadále věnuje zkoumání schopnosti abstrakce člověka vzhledem k jeho věku a dochází k závěru, že společně s věkem schopnost abstrakce mírně klesá. Zmiňuje také, že vztah inteligence jedince a schopnost jeho abstrakce je neměnný po celý život. Silná závislost schopnosti abstrakce jedince na jeho inteligenci se také potvrdila v této disertační práci.

Na základě svého výzkumu docházím k přesvědčení, že užití podobných metod by mohlo být prospěšné také na vysokých školách, zejména pak pro studenty učitelských studijních oborů.

Papers included in the thesis

1. CHYTRÝ, Vlastimil. Korelace logického myšlení a inteligence s vybranými matematickými hrami. In: *Dva dny s didaktikou matematiky*. 1. vyd. Praha: Katedra matematiky a didaktiky matematiky PF UK, 2012, s. 102-106. ISBN 978-80-7290-604-
2. CHYTRÝ, Vlastimil. Games in the maths education. In: *Mathematica IV*. 1. vyd. Ružomberok: Catholic university, 2012, s. 69-74. ISBN 978-80-8084-954-2.
3. MALINOVÁ, Dagmar a Vlastimil CHYTRÝ. Manipulace jako významný aspekt při motivaci žáků v matematice. In: *Motivace nadaných žáků a studentů v matematice a přírodních vědách*. 1. vyd. Brno, 2012. ISBN 978-80-210-5886-6.
4. CHYTRÝ, Vlastimil. Logické myšlení jedince a jeho rozvoj pomocí matematických her. In: *Matematika 5*. 1. vyd. Olomouc, 2012, s. 93 – 97. ISBN 978 – 80 – 244 – 3048 – 5.
5. MALINOVÁ, Dagmar a Vlastimil CHYTRÝ. Manipulace jako významný aspekt při motivaci žáků v matematice. In: NOVOTNÁ, Jarmila. *Motivace nadaných žáků a studentů v matematice a přírodních vědách.* Brno: Masarykova univerzita, 2013, s. 152-168. ISBN 978-80-210-6144-6.

Reference

1. ALBRECHT, Karl. *Brain building: easy games to develop your problem-solving skills*. 1. Owl Books Ed. Englewood Cliffs, N.J.: Prentice-Hall, c1984, xiii, 92 p. ISBN 01-308-1034-7.

2. DUNLAP, John. *Mathematical Thinking* [online]. c2001, poslední revize 15. 10. 2010 [cit 2011 – 06 – 03]. Dostupné z: < http://ctzalehamnmathematicalthinking.blogspot.cz/2010/10/mthematical-thinking.html>.
3. GRAHAM, Karen, Albert CUOCO a Gwen ZIMMERMANN. *Focus in high school mathematics: a formal approach*. 1. Aufl. Reston, VA: National Council of Teachers of Mathematics, c2010, ix, 78 p. Cognitive technologies. ISBN 08-735-3640-1.

4. HARTL, Pavel a Helena HARTLOVÁ. *Velký psychologický slovník*. Vyd. 4., V Portálu 1. Ilustrace Karel Nepraš. Praha: Portál, 2010, 797 s. ISBN 978-80-7367-686-5.

5. LABOUVIE-VIEF, G. (1992) A neo-Piagetian perspective on adult cognitive development. In R.J. STERNBERG & C.A BERG (eds) *Intellectual Development*, New York, Cambridge University, Press.

6. LEE, Jason S. *Abstraction and aging: a social psychological analysis*. New York [u.a.]: Springer, 1990. ISBN 978-038-7974-330.

7. PEREGRIN, Jaroslav. *Logika a logiky: systém klasické výrokové logiky, jeho rozšíření a alternativy*. Vyd. 1. Praha: Academia, 2004, 205 s. ISBN 80-200-1187-0.

8. PETERS, Sally. Playing Games and Learning Mathematics: The results of Two Intervention Studie. *International Journal of Early Years Education*. 1998, roč. 6, č. 1, s. 49 – 58.

Supplements

Supplement 1. The Entry Test of Logical Thinking

Name and surname ……..

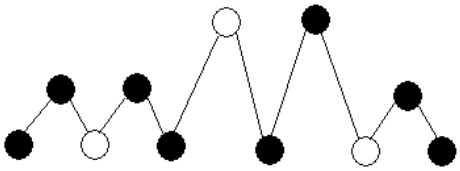
Class ……

School ……

Mark in Mathematics …….

1) Complete the numeric series by at least three more numbers.
   a) 1, 2, 3, 5, 8, 13, 21……
   b) 1, 2, 3, 6, 11, 20……
   c) 1, 3, 2, 4, 5, 7, 6, 8……

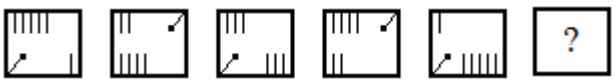
2) Draw the continuation of the following pictures. Add at least three more moves.



3) The answer to number 52363 is 36325; what is the answer to number 46251? Circle.

A) 25641
B) 26451
C) 12654
D) 51462
E) 15264

4) Which of the pictures completes best the series? Circle.



A)  B)  C)  D) 

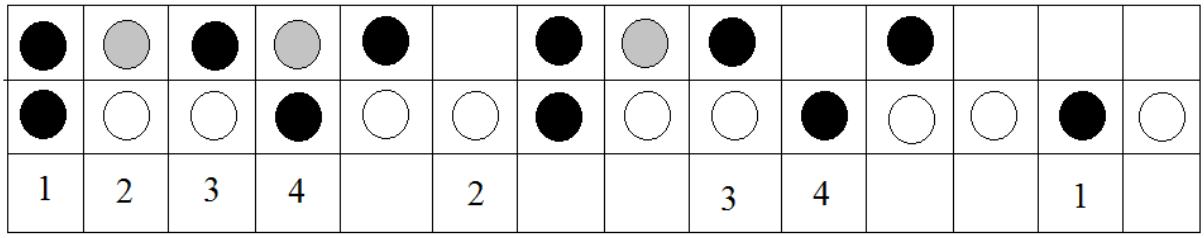
5) Substitute the question marks in the tables by numbers corresponding to the numbers already present.

1	9	**?**	13
2	10	6	14
?	**?**	7	15
4	12	8	**?**

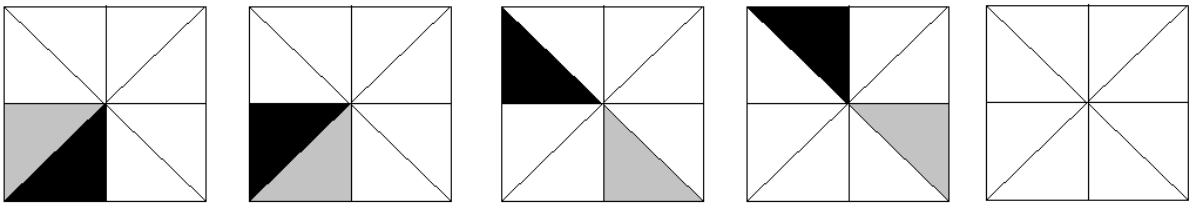
1	4	9	**?**	25	36	**?**

6) Find the regularity and fill in all the empty fields.

a)



b) In the last square, paint in the grey and the black fields.



7) Logical links

a) The natural numbers are 1, 2, 3, 4 etc.

Write down the first five natural numbers divisible by three and four at a time.

……...

b) Write down the first five natural numbers divisible by three or four.

…….....

c) Is the following statement true? If a number is divisible by six, it is even. ANO - NE

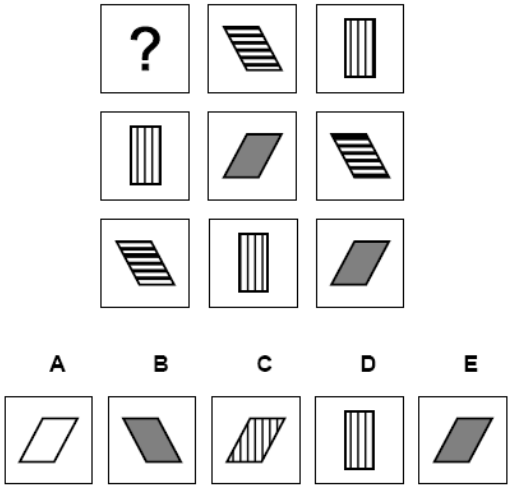
d) Is the statement reversed to the previous one true? ANO - NE

8) Write down the numbers that are even, and fall into the interval between three and sixteen, included.

……

…….

9) Choose which picture belongs to the field with the question mark (circle).



10) Answer to the following statements. Base your replies on the expressions used in these satements.

a) If you know that each mammal drinks milk and a dolphin is a mammal, what conclusion do you draw from that? .....
…….....

b) Let´s take this statement: Each child has at least one friend. When is this statement not right?

.....

.....

11) We keep three dogs in the flat, and each dog has its own bed. Alik lies in Bertik´s bed and Rex is not in his own. In which bed is Bertik?

.....

.....

12) If I do the homework and the training is cancelled, I will go to see my friend. What does the fact mean that I did not go to see my friend? Answer in full sentences. In your reply make use namely of the expressions used in the assignment.

……

……

……

Supplement 2. The Sudoku Answer Sheet

Name, surname…….. Do you know the rules of Sudoku? .....

Age……... How often do you play Sudoku?.....

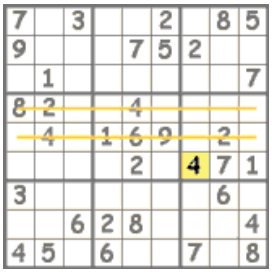
Mark in Mathematics……

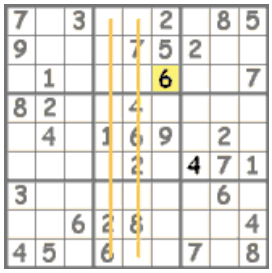
How to play Sudoku?

a) On each line a particular number only occurs once.

b) In each column a particular number only occurs once.

c) In each square of nine fields a particular number only occurs once.





Show how you manage to fill in all the fields

Easy

			2	6		7		1
6	8			7			9	
1	9				4	5		
8	2		1				4	
		4	6		2	9		
	5				3		2	8
		9	3				7	4
	4			5			3	6
7		3		1	8			

Easy

1			4	8	9			6
7	3						4	
					1	2	9	5
		7	1	2		6		
5			7		3			8
		6		9	5	7		
9	1	4	6					
	2						3	7
8			5	1	2			4

Medium difficulty

	2		6		8			
5	8				9	7		
				4				
3	7					5		
6								4
		8					1	3
				2				
		9	8				3	6
			3		6		9	

Difficult

			6			4		
7					3	6		
				9	1		8	
	5		1	8				3
			3		6		4	5
	4		2				6	
9		3						
	2					1		

In what order and time did you fill in the individual fields?

			2	6		7		1	i
6	8			7			9		h
1	9				4	5			g
8	2		1				4		f
		4	6		2	9			e
	5				3		2	8	d
		9	3				7	4	c
	4			5			3	6	b
7		3		1	8				a

A B C D E F G H I

During the game, complete into this table the order in which you were inserting the numbers (for example Ai and likewise).

1.		10.		19.		28.		37.	
2.		11.		20.		29.		38.	
3.		12.		21.		30.		39.	
4.		13.		22.		31.		40.	
5.		14.		23.		32.		41.	
6.		15.		14.		33.		42.	
7.		16.		25.		34.		43.	
8.		17.		26.		35.		44.	
9.		18.		27.		36.		45.	

Describe how you uncovered the first three fields which you completed. Do not guess, but explain why you wrote down the actual number you did.

……..

……..

……..

……..